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Height-Diameter Relationships for Conifer Species on the Blacks Mountain Experimental Forest

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An equation is presented for predicting total height as a function of diameter outside bark at breast height for conifer species of the eastside pine type on the Blacks Mountain Experimental Forest in northeastern California. Weighted nonlinear regression was used to estimate the equation coefficients. The equation, along with the species-specific regression coefficients, provides a reliable basis for estimating missing heights on inventory and growth plots.

Retrieval Terms: height estimation, ponderosa pine, Jeffrey pine, California white fir, incense-cedar

Note: Contribution #1 of the Blacks Mountain Interdisciplinary Research Project, Pacific Southwest Research Station, USDA Forest Service.

Tree diameter at breast height (dbh) can be measured rapidly and accurately. It is therefore standard procedure to measure dbh of all trees on growth and inventory plots. On the other hand, tree heights are time-consuming to measure accurately, and standard procedure is to measure only a subsample of heights or not measure them at all. However, some estimate of tree height is needed for volume calculations and for estimation of growth by forest growth and yield projection systems. When tree heights are missing in the inventory data, missing values may be estimated if the relationship of dbh to total tree height is known.

The purpose of this note is to present an equation, along with species-dependent regression coefficients, that expresses the height-diameter relationship for pine (ponderosa and Jeffrey), California white fir (*Abies concolor* var. *lowiana* [Gord.] Lemm.), and incense-cedar (*Libocedrus decurrens* Torr.)—the major coniferous species at Blacks Mountain Experimental Forest (BMEF) and throughout northeastern California. Because of the apparent similarity of their silvical characteristics and because they grow at about the same rate, no distinction is usually made between ponderosa pine (*Pinus ponderosa* Dougl. ex Laws. var. *ponderosa*) and Jeffrey pine (*P. jeffreyi* Grev. & Balf.) for growth and yield purposes.[1] These two species are combined for this analysis and are referred to collectively as "pine."

METHODS

Data Description

Total height-dbh data were collected in 1990 and 1991 during the remeasurement of 37 permanent plots located on BMEF. The plots were installed nearly 50 years ago to compare several methods of harvest cutting in the eastside pine type of northeastern California.[2] Each of these methods-of-cutting plots is 20 acres[3] in size. The outside bark diameter of all trees greater than 11.5 inches dbh was measured to the nearest 0.1 inch with a diameter tape. Total height of every 10th tree encountered was measured using the percent scale of a clinometer at a measured horizontal distance from the base of the tree. Total heights were calculated to the nearest foot.

Four 1-acre strips, each 1 chain wide (1 chain = 66 feet) and 10 chains long, were selected at random in each plot to sample the pole-sized trees (3.6 to 11.5 inches dbh). All poles in the strips were tallied by species and 2-inch diameter classes. Actual diameter to the nearest 0.1 inch and total height to the nearest foot were measured on every 10th pole tree tallied. Heights of trees less than 30 feet were measured directly with a telescoping fiberglass pole; taller poles were measured indirectly with the clinometer.

The sample-tree data represented a wide range of diameters and heights for all the conifer species on the permanent plots (*table 1, fig. 1*). For this analysis it was assumed that this sampling scheme was

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Table 1—*Number and range of measurements of sample trees*

Species	Sample size	Diameter		Height	
		Mean	Range	Mean	Range
		inches		*feet*	
Pine (ponderosa and Jeffrey)	2844	14.2	3.6-53.7	57	9-147
White fir	1313	11.4	3.6-40.3	50	13-116
Incense-cedar	302	14.6	3.6-50.2	43	11-111

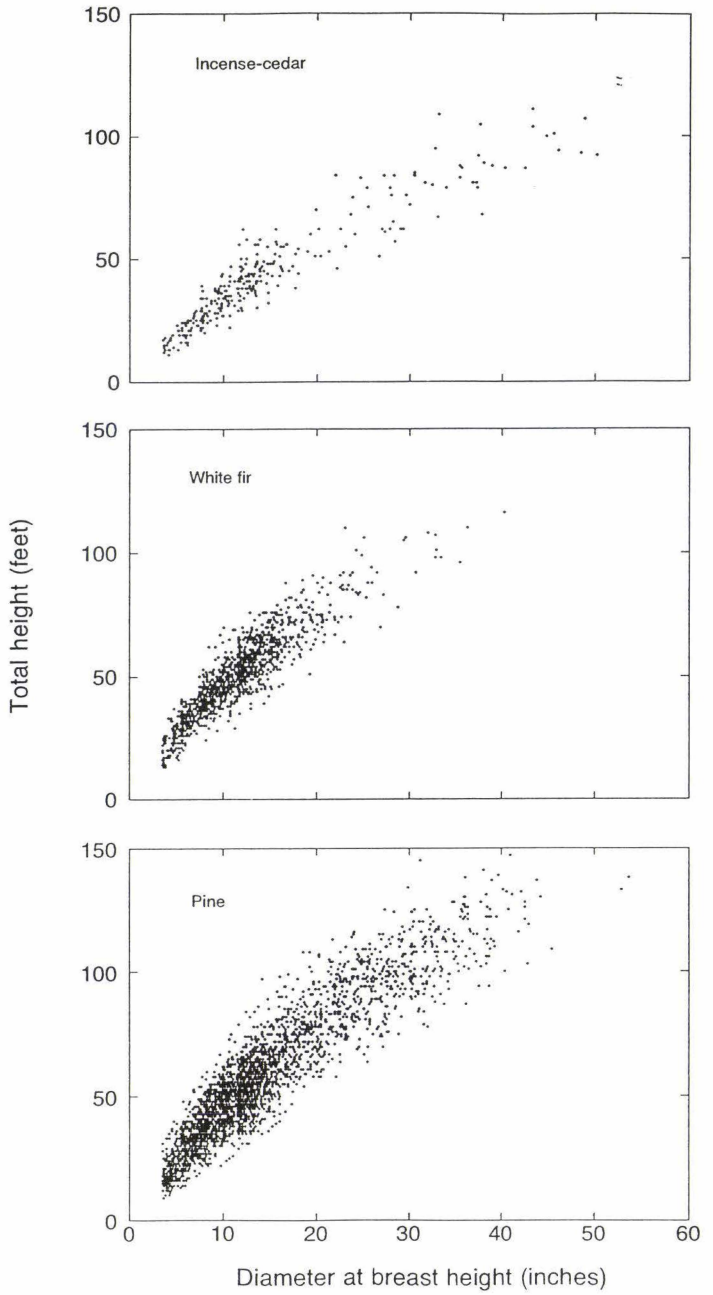

Figure 1—Total tree height plotted over diameter outside bark at breast height for the conifer species on Blacks Mountain Experimental Forest.

equivalent to a randomized design because every 10th tree was measured for total height and the sample trees were selected in an unbiased manner. Trees with broken or severely deformed tops were not included in the data set.

Data Analysis

The linear and log-linear height-diameter equations suggested by Curtis[4] may produce unreasonable estimates for small trees or introduce problems with log bias when using the logarithm of height as the dependent variable. Therefore, two nonlinear equation forms which have been recommended for conifer species in the western United States were tested to see which would best describe the height-diameter relationships at Blacks Mountain. The first equation tested,

$$\text{Ht} = 4.5 + \exp\left(b_0 + \left(\frac{b_1}{\text{dbh} + 1}\right)\right) + \text{e} \quad (1),$$

in which

Ht = total tree height in feet
dbh = diameter at breast height (outside bark) in inches
b = regression coefficients
e = a random error component with an expected value of zero and variance of σ^2,

was proposed by Wykoff and others[5] for several conifer species of the northern Rocky Mountains. The second equation considered,

$$\text{Ht} = 4.5 + \exp\left(b_0 + b_1 \times \text{dbh}^{b_2}\right) + \text{e} \quad (2),$$

was used by Larsen and Hann[6] for seven conifer species in southwest Oregon and by Dolph[7] for young-growth red fir (*Abies magnifica* A. Murr.) in California and southern Oregon.

Equations 1 and 2 were each fitted to the data by nonlinear regression. An examination of the residuals from preliminary fits of the equations to the data revealed that the variance was not homogeneous but increased with increasing values of dbh, indicating weighted regression was needed to homogenize the variance for each species. Both equations were then fitted to the data by weighted nonlinear regression. Weights of 1/dbh, 1/dbh^2, 1/dbh^4, and 1.0 (un-weighted) were tried. Because equation 2 consistently

resulted in the smallest residual mean square for each species and for each weight (*table 2*), it was chosen as the basic equation form. Using equation 2, the four weighting procedures were then compared using Furnival's index of fit.[8]

RESULTS AND DISCUSSION

The studies by both Larsen and Hann and by Dolph found that more accurate and precise predictions of tree height could be made when the variables of site index and basal area were included in an expanded form of equation 2. However, the site index and basal area associated with individual trees were not available for the development of the height-diameter relationships for the BMEF conifers. Even if site index information had been available, the range of site qualities is so narrow on the experimental forest that the site variable would probably not have been significant. Site index for ponderosa pine at base age of 100 years varies only from 60 to 80 feet and averages 72 feet at BMEF.[9]

Furnival's index of fit indicated that 1/dbh provided the best weight for equation 2 for each of the Blacks Mountain conifer species (*table 3*). Weighted residuals were calculated by subtracting predicted weighted height from observed weighted height. The effect of weighting on the standard deviation of the residuals was demonstrated by grouping the data into 4-inch diameter classes and calculating the standard deviation of both the unweighted and the weighted residuals. The plot of the standard deviations of the weighted and unweighted residuals for each 4-inch dbh class showed this weighting procedure did homogenize the variance for each species (*fig. 2*).

Regression coefficients for equation 2 were estimated using the secant, or DUD, method of the SAS NLIN procedure.[10] Using these coefficients (*table 4*), the predicted heights ($\widehat{\text{Ht}}$) for each species were calculated by 2-inch dbh intervals over the range of diameters found on the sample plots (*table 5*). The 95 percent confidence and prediction intervals around the value of $\widehat{\text{Ht}}$ were then calculated at each 2-inch interval of dbh from 0 to 60 inches (*fig. 3*). The confidence interval of mean Ht for a specified dbh is given by

Table 2—*Residual mean square for equations 1 and 2 for each species and each weighting procedure*

Species	Unweighted	Weights 1/dbh	Weights 1/dbh²	Weights 1/dbh⁴
	residual mean square			
Pine				
Equation 1	97.995	7.393	0.749	0.014
Equation 2	86.720	6.515	0.633	0.012
White fir				
Equation 1	47.413	4.073	0.421	0.008
Equation 2	42.562	3.785	0.404	0.008
Incense-cedar				
Equation 1	53.545	3.520	0.323	0.006
Equation 2	45.738	2.956	0.251	0.004

Table 3—*Furnival's index of fit for each species and each weighting procedure used with equation 2*

Species	Unweighted	Weights 1/dbh	Weights 1/dbh²	Weights 1/dbh⁴
	index of fit			
Pine	9.31	8.97	9.81	16.66
White fir	6.52	6.31	6.68	10.06
Incense-cedar	6.76	5.99	6.09	9.00

$$\widehat{\text{Ht}} \pm z \cdot \sqrt{\text{Var}\widehat{\text{Ht}}} \cdot \sqrt{\text{dbh}} \qquad (3),$$

and the prediction interval for an individual value of $\widehat{Ht}$ at a specified dbh is

$$\widehat{\text{Ht}} \pm z \cdot \sqrt{\text{Var}\widehat{\text{Ht}} + \widehat{\sigma^2}} \cdot \sqrt{\text{dbh}}, \qquad (4),$$

in which

$\text{Var}\widehat{\text{Ht}}$ = estimated variance of $\widehat{Ht}$ at a specified dbh as calculated using the Taylor approximation (delta method)[11]

$\widehat{\sigma}^2$ = the residual mean square from the weighted nonlinear least squares regression.

z = Standard Normal-z cut-off point

Estimated total heights for white fir are taller than those for pine for diameters less than about 14 inches (*table 5*). For diameters 14 inches and larger, the estimated heights for white fir are substantially less than those of pine. This appears to happen because sites are too hot and dry at lower elevations on the experimental forest for white fir to prosper in a dominant position. Although it grows rapidly in the understory, when it reaches the canopy, growth rates decline and tops are often killed.

Foresters often need estimates of total tree heights when only the diameters of the trees are known. Equation 2, along with the species-specific regression coefficients (*table 4*), provides a simple and consistent method of estimating missing heights on growth and inventory plots on the Blacks Mountain Experimental Forest. If the equation is used to estimate heights of trees larger than those in the data base, or if it is used outside the Blacks Mountain Experimental Forest, the reliability of such estimates will be unknown since they would be made with extrapolations of the basic data. Heights of trees with broken, dead, or severely deformed tops should not be estimated with this equation.

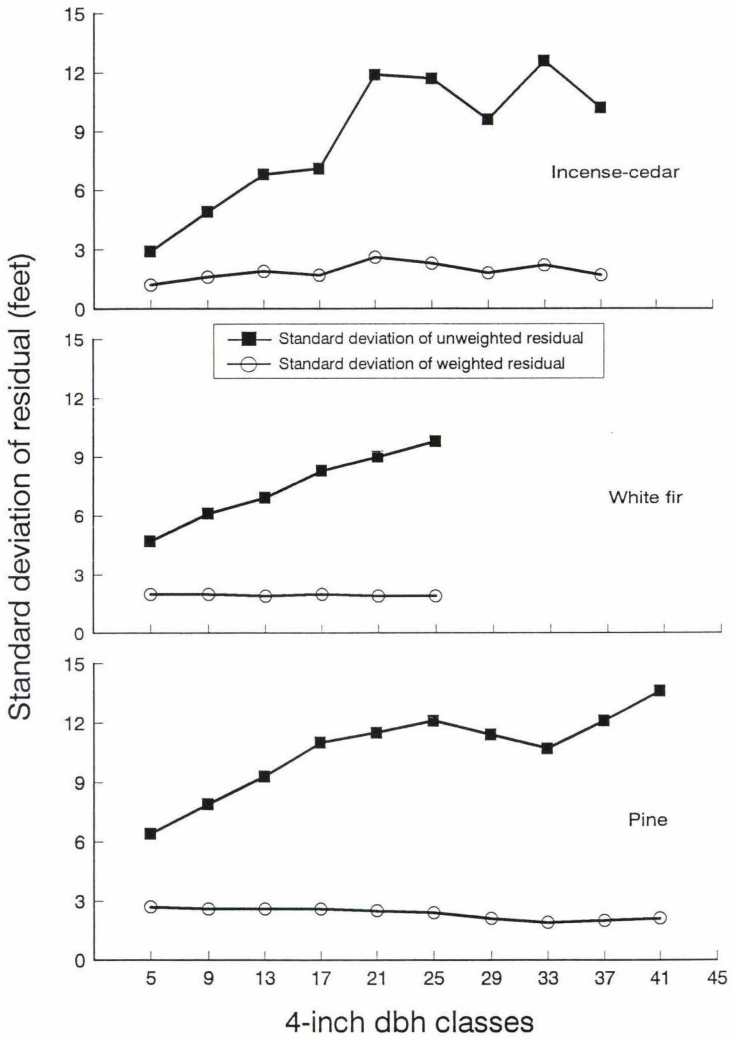

Figure 2—The effect of weighting on the standard deviation of the residuals by 4-inch diameter classes (weight = 1/dbh).

Table 4—*Regression coefficients for equation 2 estimated with the error variance weighted by 1/dbh*

Species	Coefficients		
	b_0	b_1	b_2
Pine	6.5025	-6.1361	-0.341
White fir	6.1876	-5.3296	-0.3382
Incense-cedar	6.0191	-6.2000	-0.3722

Table 5—*Total height estimated using equation 2 and the regression coefficients in Table 4 for the range of diameters for each species in the data base*

Diameter	Estimated total height		
	Pine	White fir	Incense cedar
--inches--	---------- *feet* ----------		
4.0	19.1	21.8	14.7
6.0	28.4	31.1	21.6
8.0	37.1	39.3	28.1
10.0	45.1	46.7	34.1
12.0	52.6	53.3	39.7
14.0	59.5	59.3	44.8
16.0	66.0	64.9	49.7
18.0	72.1	70.0	54.1
20.0	77.8	74.8	58.3
22.0	83.1	79.2	62.3
24.0	88.2	83.4	66.0
26.0	93.0	87.3	69.6
28.0	97.6	91.0	72.9
30.0	101.9	94.6	76.1
32.0	106.1	97.9	79.1
34.0	110.1	101.1	82.0
36.0	113.9	104.1	84.8
38.0	117.6	107.0	87.4
40.0	121.1	109.8	90.0
42.0	124.5		92.4
44.0	127.8		94.8
46.0	131.0		97.1
48.0	134.1		99.3
50.0	137.0		101.4
52.0	140.0		
54.0	142.7		

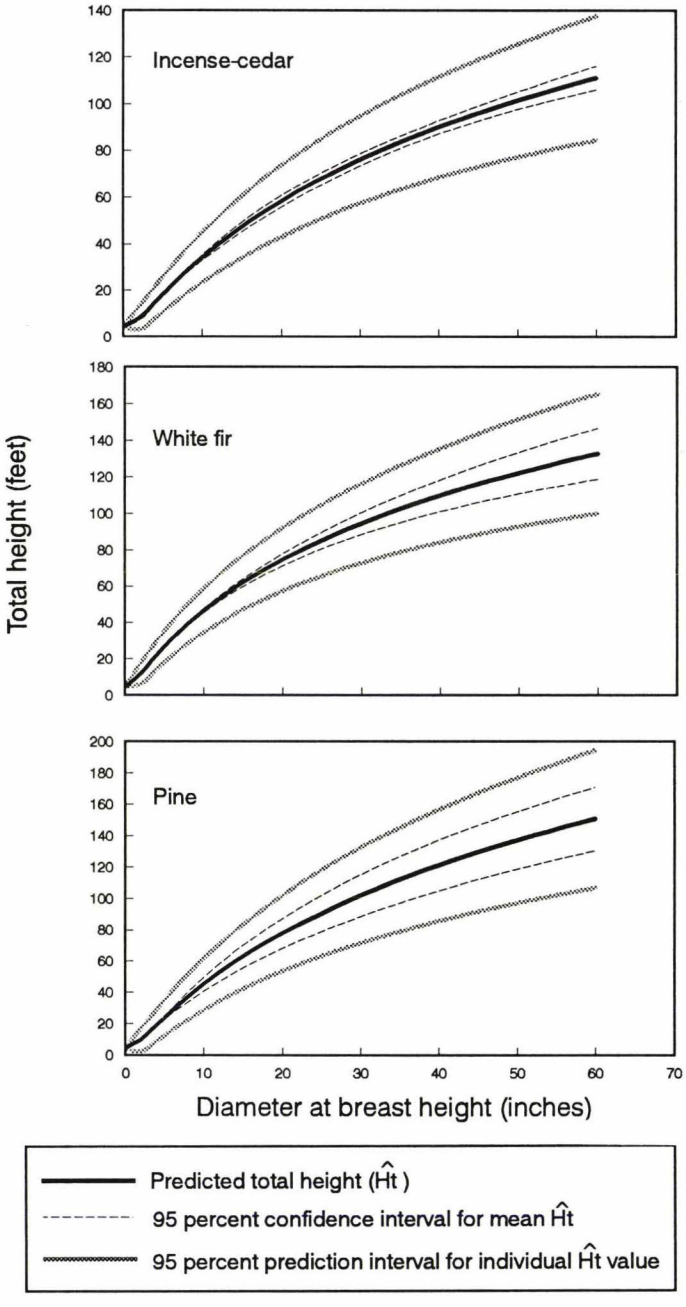

Figure 3—Total height predicted by equation 2 plotted over diameter at breast height and the 95 percent confidence and prediction intervals for each of the conifer species at Blacks Mountain Experimental Forest.

END NOTES AND REFERENCES

[1]Hallin, William E. 1957. ***Silvical characteristics of Jeffrey pine***. Tech. Paper 17. Berkeley, CA: California Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 11 p.

[2]Hasel, A. A. 1938. ***Plan for methods-of-cutting study, Blacks Mountain sustained yield project***. Unpublished Manuscript, Copy on file at Pacific Southwest Research Station, Forest Service, U.S. Department of Agriculture, Redding Silviculture Laboratory, Redding, CA.

[3]British/Metric Conversion

1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 acre = 0.404686 hectares

[4]Curtis, Robert O. 1967. ***Height-diameter and height-diameter-age equations for second-growth Douglas-fir***. Forest Science 13(4): 365-375.

[5]Wykoff, William R.; Crookston, Nicholas L.; Stage, Albert R. 1982. ***User's guide to the stand prognosis model***. Gen. Tech. Rep. INT-133. Ogden, UT: Intermountain Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 112 p.

[6]Larsen, David R.; Hann, David W. 1987. ***Height-diameter equations for seventeen tree species in southwest Oregon***. Res. Paper 49. Corvallis, OR: Forest Research Laboratory, Oregon State University; 16 p.

[7]Dolph, K. Leroy. 1989. ***Height-diameter equations for young-growth red fir in California and southern Oregon***. Res. Note PSW-408. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 4 p.

[8]Furnival, George M. 1961. ***An index for comparing equations used in constructing volume equations***. Forest Science 7(4): 337-341.

[9]Hallin, William E. 1959. ***The application of unit area control in the management of ponderosa-Jeffrey pine at Blacks Mountain Experimental Forest***. Tech. Bull. 1191. Washington, DC: U. S. Department of Agriculture; 96 p.

[10]SAS Institute Inc. 1982. ***SAS user's guide: statistics, 1982 edition***. Cary, NC: SAS Institute Inc; 584 p.

[11]Bishop, Yvonne M. M.; Fienberg, Stephen E.; Holland, Paul W. 1975. ***Discrete multivariate analysis; theory and practice***. Cambridge, MA: MIT Press; 486-488.

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